PLASTIC DEFORMATION OF A ROUGH SURFACE

M. U. Ushitskii

UDC 539.214

In certain technological problems and, in particular, for the production of heterogeneous composition materials by the joining of their components in the solid state, an important consideration is the process of formation of physical contact between them, which is accompanied by plastic deformation of the microrelief at the joined surfaces [1].

The basic model problem of the formation of physical contact is the problem of plastic deformation of a rough surface of a perfectly plastic body by a smooth rigid stamp [2]. We discuss the solution of this problem under conditions of plane deformation, treating the profile H(x) of the rough surface as a realization of a stationary random process.

Following Demkin [2], we approximate the microrelief of the deformable surface by a system of wedges. Let N such wedge-shaped microasperities be situated on a portion of a profile realization of length L. The process of crumpling of each of them can be described within the context of the self-similar solution of Hill [3], according to which the area h_i of the resulting contact plateau is proportional to the subsidence c of the stamp and is equal to

$$h_i = 2c\varphi_i \quad (i = 1, 2, \ldots, N), \tag{1}$$

where $\varphi_i = (1 + \sin\psi_i)\sec\psi_i$ and ψ_i is the angle of the centered fan of the field of slip lines. Here the force p_i acting on the wedge is [3]

$$p_i = \sigma_c h_i = 4kc\chi_i \quad (i = 1, 2, \ldots, N), \tag{2}$$

where $\sigma_c = 2k(1 + \psi_i)$ is the contact stress determined by Hill's solution [3], k is the yield point in shear, and $\chi_i = (1 + \psi_i)(1 + \sin\psi_i)\sec\psi_i$.

We use relations (1) and (2) to construct equations describing the crumpling of the ensemble of wedge-shaped microasperities. We enumerate the heights H_i of the tips of the microasperities on the realization segment L in increasing order from H_1 to H_N and fix the initial position of the stamp at height H_N. Moving the stamp downward in arbitrarily small increments in such a way that each increment of subsidence corresponds to the crumpling of at most one new microasperity, we determine the total contact area h. For this purpose we partition the interval of variation of the subsidence c of the stamp into N subintervals as follows:

$$\begin{gathered} 0 < c_1 \leqslant H_N - H_{N-1}, \\ H_N - H_{N-1} < c_2 \leqslant H_N - H_{N-2}, \\ \cdots \\ H_N - H_{N-m+1} < c_m \leqslant H_N - H_{N-m}, \\ \cdots \\ H_N - H_1 < c_N. \end{gathered} \tag{3}$$

Suppose that in each of these subintervals the stamp moves through n_i ($i = 1, 2, \ldots, N$) successive steps, so that for the l-th subsidence step in the m-th subinterval ($l \leqslant n_m$) the subsidence $c_{m,l}$ can be written in the form

$$c_{m,l} = \sum_{j=1}^{n_1} c_{1,j} + \sum_{j=1}^{n_2} c_{2,j} + \ldots + \sum_{j=1}^{n_{m-1}} c_{m-1,j} + \sum_{j=1}^{l} c_{m,j} = \sum_{i=1}^{m-1}\sum_{j=1}^{n_i} c_{i,j} + \sum_{j=1}^{l} c_{m,j}.$$

The total contact plateau for any fixed position of the stamp in the subintervals (3) is determined with regard for (1) by the terms of the sequence

$$\begin{aligned} &h = h_N = 2c_{1,l}\varphi_N, \\ &h = h_N + h_{N-1} = 2c_{2,l}\varphi_N + 2\,[c_{2,l} - (H_N - H_{N-1})]\,\varphi_{N-1}, \\ &\cdots \\ &h = \sum_{i=N-m+1}^{N} h_i = 2c_{m,l}\varphi_N + 2\,[c_{m,l} - (H_N - H_{N-1})]\times \end{aligned}$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 183-187, July-August, 1978. Original article submitted July 7, 1977.

 0021-8944/78/1904-0572$07.50

$$\times\varphi_{N-1}+\ldots+2\left[c_{m,l}-(H_N-H_{N-m+1})\right]\varphi_{N-m+1},$$

$$\cdots$$

$$h=\sum_{i=1}^{N}h_i=2c_{N,l}\varphi_N+2\left[c_{N,l}-(H_N-H_{N-1})\right]\varphi_{N-1}+\ldots 2\left[c_{N,l}-(H_N-H_1)\right]\varphi_1. \tag{4}$$

On the basis of relation (2) an exactly analogous sequence can be formed, describing the growth of the contact force in each increment of subsidence of the stamp:

$$p=p_N=4kc_{1,l}\chi_N,$$

$$p=p_N+p_{N-1}=4kc_{2,l}\chi_N+4k\left[c_{2,l}-(H_N-H_{N-1})\right]\chi_{N-1},$$

$$\cdots$$

$$p=\sum_{i=N-m+1}^{N}p_i=4kc_{m,l}\chi_N+4k\left[c_{m,l}-(H_N-H_{N-1})\right]\chi_{N-1}+\ldots+4k\left[c_{m,l}-(H_N-H_{N-m+1})\right]\chi_{N-m+1},$$

$$\cdots \tag{5}$$

$$p=\sum_{i=1}^{N}p_i=4kc_{N,l}\chi_N+4k\left[c_{N,l}-(H_N-H_{N-1})\right]\times$$

$$\times\chi_{N-1}+\ldots+4k\left[c_{N,l}-(H_N-H_{N-1})\right]\chi_1.$$

We transform the m-th terms of the sequences (4) and (5) as follows:

$$h_{m,l}=2c_{m,l}\sum_{i=N-m+1}^{N}\varphi_i-2\sum_{i=N-m+1}^{N-1}(H_N-H_i)\varphi_i,$$

$$p_{m,l}=4kc_{m,l}\sum_{i=N-m+1}^{N}\chi_i-4k\sum_{i=N-m+1}^{N-1}(H_N-H_i)\chi_i$$

and calculate the increments of the area of contact Δh_m and force Δp_m in the l-th step of the m-th subsidence subinterval:

$$\Delta h_m=h_{m,l}-h_{m,l-1}=2\Delta c\sum_{i=N-m}^{N}\varphi_i,$$

$$\Delta p_m=p_{m,l}-p_{m,l-1}=4k\Delta c\sum_{i=N-m}^{N}\chi_i. \tag{6}$$

Inasmuch as $\sum_{i=N-m}^{N}\varphi_i=m\bar{\varphi}$ and $\sum_{i=N-m}^{N}\chi_i=m\bar{\chi}$, relation (6) can be written

$$\Delta h_m/\Delta c=2m\bar{\varphi},\quad \Delta p_m/\Delta c=4km\bar{\chi}, \tag{7}$$

where $\bar{\varphi}$ and $\bar{\chi}$ are the average values over m of the random variables φ and χ.

Passing to the limit $\Delta c\to 0$ in Eqs. (7), we obtain differential equations describing the growth of the contact area and contact force in the subinterval $[H_N-H_m;\ H_N-H_{N-m+1}]$:

$$dh_m/dc=2\bar{\varphi}m,\quad dp_m/dc=4k\bar{\chi}m, \tag{8}$$

where m is the number of asperity tips crumpled by the stamp when it is in the position such that c = C.

Using the average-value representations of h and p, we integrate Eqs. (8). We obtain expressions for the average values of the unit contact area $\bar{h}$ and unit contact force $\bar{p}$. Using the initial conditions $\bar{h}(c=0)=0$, $\bar{p}(c=0)=0$ and the continuity conditions for $\bar{h}(c)$ and $\bar{p}(c)$ at the endpoints of the subintervals (3), we can represent these solutions in the form

$$\bar{h}_m(c)=2\bar{\varphi}\frac{m}{N}c-2\bar{\varphi}\frac{1}{N}\sum_{i=1}^{m-1}(H_N-H_{N-i}),$$

$$\bar{p}_m(c)=4k\bar{\chi}\frac{m}{N}c-4k\bar{\chi}\frac{1}{N}\sum_{i=1}^{m-1}(H_N-H_{N-i}). \tag{9}$$

The ratio m/N represents the approximate value of the probability of crumpling of the m-th microasperity when c = C. We write the difference (H_N-H_{N-i}) entering into the solutions (9) as follows:

$$H_N-H_{N-i}=(H_N-H_{N-1})+(H_{N-1}-H_{N-2})+\ldots+(H_{N-i+1}-H_{N-i}). \tag{10}$$

We use expression (10) to represent relations (9) in the form

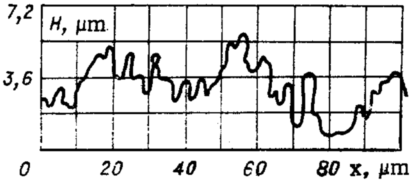

Fig. 1

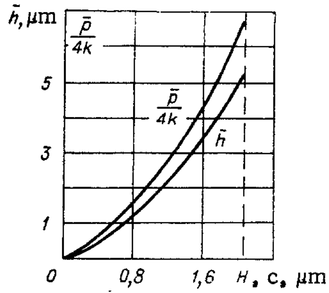

Fig. 2

$$\bar{h}_m(c) = 2\bar{\varphi}\left[\frac{m}{N}c - \sum_{i=1}^{m-1}\frac{i}{N}\Delta H_i\right],$$
$$\bar{p}_m(c) = 4k\bar{\chi}\left[\frac{m}{N}c - \sum_{i=1}^{m-1}\frac{i}{N}\Delta H_i\right], \tag{11}$$

where $\Delta H_i = H_{n-i+1} - H_{N-i}$.

Postulating the existence of a continuous distribution of asperity tips of the profile $f(H)$ and passing to the limit $N \to \infty$ in relations (11), we obtain asymptotic expressions for $\bar{h}(c)$ and $\bar{p}(c)$:

$$\bar{h}(c) = 2\bar{\varphi}\left[c\int_{H_N-c}^{H_N} f(H)\,dH - \int_{H_N-c}^{H_N}\int_{H_N-c=H}^{H_N} f(H)\,dH\,dH\right],$$
$$\bar{p}(c) = 4k\bar{\chi}\left[c\int_{H_N-c}^{H_N} f(H)\,dH - \int_{H_N-c}^{H_N}\int_{H_N-c=H}^{H_N} f(H)\,dH\,dH\right]. \tag{12}$$

For a perfectly plastic incompressible material, on the basis of the incompressibility condition it is necessary to limit the subsidence of the stamp to a value C_{1m}:

$$c \leqslant C_{1m} = \bar{H}, \tag{13}$$

Where $\bar{H}$ is the arithmetic-mean height of the profile $H(x)$. Thus, the solutions obtained above are defined in the interval $0 \leqslant c \leqslant \bar{H}$.

For the construction of the solutions (12) it is necessary to determine $\bar{\chi}$, $\bar{\varphi}$, and $f(H)$. The average values of the random variables φ and χ are calculated by the usual methods of mathematical statistics, and for the determination of the function $f(H)$ two approaches are possible: 1) a theoretical approach based on the Cramér limit theorem [4]; 2) an empirical approach using statistical methods. According to Hudson [5], the Cramér limit theorem for stationary Gaussian random processes can be used to approximate m by the average number of maxima of the function $H(x)$ situated in a unit interval and exceeding the level $H_N - c$:

$$m = \frac{1}{2\pi}\sqrt{-\frac{K^{(4)}(0)}{K''(0)}}\left\{1 - \Phi\left(\frac{H_N - c}{\sqrt{K(0)}}\right) - \frac{\rho}{2}\exp\left[-\frac{(H_N - c)^2}{2K(0)}\right] - \right.$$
$$\left. - \sqrt{2\pi}\sum_{\nu=1}^{\infty}\frac{\rho(2\nu)}{(2\nu)!}\Phi(2\nu)\left(\frac{H_N - c}{\sqrt{K(0)}}\right)\Phi(2\nu - 1)(0)\right\}, \tag{14}$$

where $K(x)$ is the correlation function of the profile $H(x)$,

$$\rho = \frac{K''(0)}{K(0)K^{(4)}(0)}; \quad \Phi(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left(-\frac{v^2}{2}\right)dv$$

is the probability integral.

The average number N of maxima in a unit interval of a stationary Gaussian random process is defined as follows [6, 4]:

$$N = \frac{1}{2\pi}\sqrt{-\frac{K^{(4)}(0)}{K''(0)}}. \tag{15}$$

Using (15) and (14), we obtain the following expression for the probability of crumpling of the m-th asperity tip:

$$\frac{m}{N}=1-\Phi\left(\frac{H_N-c}{\sqrt{K(0)}}\right)-\frac{\rho}{2}\exp\left[-\frac{(H_N-c)^2}{2K(0)}\right]-$$
$$-\sqrt{2\pi}\sum_{\nu=1}^{\infty}\frac{\rho^{2\nu}}{(2\nu)!}\Phi(2\nu)\left(\frac{H_N-c}{\sqrt{K(0)}}\right)\Phi(2\nu-1)(0).$$

The function f(H) can also be constructed by mathematical-statistical methods in the analysis of profilograms. Here the form of the distribution function is not subjected to any constraints, but to obtain a statistically reliable distribution function it is necessary to process a larger volume of realizations of the profile H(x) than in correlation analysis.

For a profilogram of the surface of an OT4 titanium alloy sample of length L = 1.7 mm (N = 213), for which a fragment of a realization is shown in Fig. 1, the solutions of Eqs. (12), $\bar{h}(c)$ and $\bar{p}(c)$, subject to the constraints (13), are given in Fig. 2.

The plotted curves $\bar{h}(c)$ and $\bar{p}(c)$ are completely determined by the initial configuration of the profile H(x) and agree quite well with the main experimental results in the region of contact of rough surfaces [2]. For example, the growth of the contact area and contact force takes place at an increasing rate, primarily as the result of entrainment of newly crumpled microasperities in the plastic flow. The influence of the growth of the individual contact areas in the initial period is inconsequential, but increases with subsidence of the stamp.

In application to the problem of producing a composition material in the solid state, the function $p(c) = \bar{p}(c)N$ represents a loading program that ensures the formation of physical contact between the joined components, and $\bar{h}(c)$ provides a means for determining the corresponding contact density.

The author is grateful to Yu. N. Rabotnov and V. D. Klyushnikov for their interest in the study.

LITERATURE CITED

1. É. S. Karakozov, Joining of Metals in the Solid Phase [in Russian], Metallurgiya, Moscow (1976).
2. N. B. Demkin, Bonding of Rough Surfaces [in Russian], Nauka, Moscow (1970).
3. W. Prager and P. G. Hodge, Jr., Theory of Perfectly Plastic Solids, Wiley, New York (1951).
4. H. Cramér and M. R. Leadbetter, Stationary and Related Stochastic Processes, Wiley, New York (1967).
5. D. Hudson, Statistics for Physicists [Russian translation], Mir, Moscow (1967).
6. A. P. Husu, Yu. R. Vitenberg, and V. A. Pal'mov, Surface Roughness: A Probability-Theoretic Approach [Russian translation], Mir, Moscow (1969).